Puzzles and Games – How to Use Them to Support the Standards

by
Bill Lombard

Come away from this session with great puzzles and games, create mathematical curiosity, and challenge your students. Your students will learn and love it!

Educational Consultant,
Author, and Teacher
(530) 243-2064
bill@tttpress.com
5885 Avery Way
Redding CA 96003
Teacher to Teacher Press
www.tttpress.com

"Building Mathematical Skill On a Foundation of Understanding"

Using Puzzles in Math Education

Puzzles are a great way to bring fun into the math classroom. Many students feel math classes are drudgery and just another class where they try to imitate the teacher. Well designed puzzles cause people to use their creativity, to think "outside the box", and use their skills and aptitudes in new and exciting ways.

Almost all my mathematical work is done in Microsoft Word or Powerpoint because they are the standards for word processing and presentation. MS Word has a Drawing module that works well. Math illustrations can be done with MS Word quite nicely once the user gets the hang of it.

I get puzzles from a variety of sources: a large collection of recreational math books, math journals, websites, other staff members, conferences, and my partner, Brad Fulton. After finding a puzzle, I look for new ways to use it in class, technology applications, relationship to math standards, and extensions.

Puzzles can be used to introduce, reinforce, and extend ideas in almost any math class. One of the joys of teaching is the ability to be creative with young minds. To capture students' minds with an intriguing puzzle that is self-motivational and then lead them to the underlying math principle is a joy to behold and an invigorating experience for a teacher.

To allow students to grow in their puzzle solving abilities, it is a good idea to use similar puzzles over a period of time so they can use skills developed in earlier puzzles to help them later on. You can have an introductory puzzle that almost everyone can solve, then move to a puzzle that's a little more challenging, finishing up with one requiring most students to stretch to reach the solution.

My ideal puzzles are those which everyone can understand, are intrinsically motivating, have a relatively few words, use some graphical or visual construct, and allow the demonstration of an underlying mathematical principle. The use of an interesting illustration can motivate many otherwise reluctant problem solvers. Teachers can also include in class any background they wish, including historical information. This paper includes some examples of these. No solutions are given since it's too much of a temptation to look at the answer.

When creating puzzles be careful to make directions very clear. It's a good idea to include examples of what is permitted and also what is not allowed as a solution. Try to use a presentation or a format that is familiar to most students.

Teachers must be enthusiastic puzzle posers and solvers for students to become good problem solvers. A good idea is to post on the classroom wall many approaches to problem solving. On the walls of my classroom in $8\text{-}^1/_2$ x 11 inch format are 11 problem solving signs: Find a pattern, Physical representation, Venn diagram, T-table, Work backwards, Solve a simpler problem, Find a formula, Use algebra, Draw a graph, Guess and Check, and Eliminate possibilities. When presenting a puzzle to students, I give them a few minutes to read the problem and think about it. Then, pointing to the Problem Solving Wall, I ask which types of problem solving strategies might be of use; this provides a good discussion and allows all students to hear suggestions from others. This often gives them insights they may not have thought of.

I often challenge my students when presenting new math material by telling them it's just another game or puzzle. Much of mathematics can be viewed and taught this way; by looking at it as a game, it becomes fun and engaging. Students can look for the structure of a problem, which is what separates arithmetic from algebra.

Hanging on the wall of my classroom are rulers and scissors; each has a number and is assigned to a specific student so management is simple. This allows me to use puzzles involving measurement and dissection. They are always available for student use and allow needed experience with physical models of a puzzle; models are always good referents when students are engaged in the problem solving experience. Manipulatives are pervasive in primary grades, but tend to disappear in many classrooms as students get older. I would like to encourage you to use manipulatives as an increased part of your curriculum, especially when introducing a new concept; it allows many students to make a visual connection to an abstract concept. Students in middle/high school are in transition from the concrete to the abstract, and any help we can give them is beneficial to their learning. Research tells us that students understand something if they see the connections to other things they know, and the measure of insight is consistent with the quantity and affinity of the connections. By providing multiple ways to look at a problem and its solution, you are building a strong classroom experience for mathematics learners.

Problem solving is one of the main reasons we teach mathematics. Because puzzles are fun and challenging, they can teach your students to enjoy and recognize the value of the methods used in problem solving. Here is what the National Council of Teachers of Mathematics has to say about problem solving (abridged excerpts from *Principles and Standards for School Mathematics, 2000*):

"Problem solving is the cornerstone of school mathematics. Students who can both develop and carry out a plan to solve a mathematical problem are exhibiting knowledge that is much deeper and more useful than simply carrying out a computation. The goal of school mathematics should be for all students to become increasingly able and willing to engage with and solve problems."

"Mathematical games can foster mathematical communication as students explain and justify their moves to one another. In addition, games can motivate students and engage them in thinking about and applying concepts and skills . . . "

"By reflecting on their solutions . . . students use a variety of mathematical skills, develop a deeper insight into the structure of mathematics, and gain a disposition toward generalizing."

"Good problem solvers realize what they know and don't know, what they are good at and not so good at; as a result they can use their time and energy wisely. They plan more carefully and more effectively and take time to check their progress periodically. These habits of mind are important not only in making students better problem solvers but also in helping students become better learners of mathematics."

"By allowing time for thinking, believing that young students can solve problems, listening carefully to their explanations, and structuring an environment that values the work that students do, teachers promote problem solving and help students make their strategies explicit."

"Instead of teaching problem solving separately, teachers should embed problems in the mathematics-content curriculum. When teachers integrate problem solving into the context of mathematical situations, students recognize the usefulness of strategies."

"Students are more likely to develop confidence and self-assurance as problem solvers in classrooms where they play a role in establishing the classroom norms and where everyone's ideas are respected and valued. These attitudes are essential if students are expected to make sense of mathematics and to take intellectual risks by raising questions, formulati9n conjectures, and offering mathematical arguments."

Metacognition, or thinking about thinking, is important as part of the problem solving process and also one of its end results. When students are encouraged to think about what they are doing and examine their process of solving a problem through teacher modeling, they have gained an important skill that they can use in future math problems and puzzles. Many students think there is only one way to solve a problem, and that all problems can be solved quickly and easily. When students experience many methods of solving one particular problem, and also realize that solving a single problem/puzzle can take an extended period of time, their puzzle solving sense of self-worth increases.

Teachers should experience less discipline problems with students because they are engrossed in the puzzle at hand. I've always said that the best lesson plans are the ones in which all you can see is the tops of the students' heads, because they are intent on their problems. Students are empowered when solving puzzles because they realize they have a chance to do something really cool, and there is a tremendous amount of satisfaction felt upon completion of the task. This internal satisfaction gained as feedback from the puzzle is much better for the student than having to rely on approval from the teacher. It takes the students to a higher level, since now they can seek out puzzles and problems for their own satisfaction and they become lifelong problem solvers.

I normally use puzzles on Fridays, with solutions given the following Monday. When students ask whether their puzzle solution is correct, they do not get an answer from me. They are told that I have amnesia, but that I will regain my memory on Monday. This tactic is used to help develop students' self-reliance and confidence in their own abilities. They are encouraged to share solutions with one another and thus to find out if their results are acceptable. I often have certain students work together to foster discussion and promote using mathematical language and problem solving terminology.

Many times during problem-solving or puzzle-solving activities my students say "This is impossible", or "I can't do it." They are then reminded that last year's students solved the puzzle, and also that I would not give them a puzzle without a solution. They are also told that when they are able to finally solve the puzzle, they are going to feel really good when they were able to get the answer by themselves. Perseverance is taught and cultivated this way. Students will be confronted with many challenges in math classes, and they need to have the ability to work with a problem over a period of time to solve it.

Real World Number Stumpers #2

24 =	H in a D
12 =	S of the Z
53 =	C in a D (with the J)
90 =	D in a R A
8 =	S on a S S
1001 =	A N
1 =	W on a U
11 =	P on a F T
29 =	D in F in a L Y
64 =	S on a C B
40 =	D and N of the G F
100 =	S in the U S S
¯40 =	N of D C = N of D F

hours in a day, signs of the zodiac, cards in a deck (with the joker), degrees in a right angle, sides on a stop sign, Arabian Nights, wheels on a unicycle, players on a football team, days in February in a leap year, squares on a chess board, days and nights of the great flood, senators in the United States Senate, number of degrees Celsius = number of degrees Fahrenheit

Hoppy

- ✓ Engaging game format
- ✓ Easily customized to fit ability level of class
- ✓ Leads students towards algebraic generalizations
- ✓ Strategic thinking
- ✓ Pattern and function relationship
- ✓ Vocabulary:
  - Finite differences
  - Graphing a parabola
  - Formulas
  - Domain
  - Range
  - Inductive vs. deductive reasoning
  - Independent vs. dependent variables
  - Quadratic relationship

Hoppy

First Step

Put markers labeled *X* and *Y* as shown below. Your goal is to exchange their places with the fewest number of moves. Allowable moves:

- Sliding a marker onto an open space next to it.
- Jumping a marker over a single marker next to it, as long as you land on an empty space. You may not jump over more than one marker.

Starting Position

X		Y

Ending Position

Y		X

Hoppy
Second Step

Put markers labeled *X* and *Y* as shown below. Your goal is to exchange their places with the fewest number of moves. Allowable moves:

- Sliding a marker onto an open space next to it.
- Jumping a marker over a single marker next to it, as long as you land on an empty space. You may not jump over more than one marker.

Starting Position

X	X		Y	Y

Ending Position

Y	Y		X	X

Hoppy

Third Step

Put markers labeled *X* and *Y* as shown below. Your goal is to exchange their places with the fewest number of moves. Allowable moves:

- Sliding a marker onto an open space next to it.
- Jumping a marker over a single marker next to it, as long as you land on an empty space. You may not jump over more than one marker.

Starting Position

X	X	X		Y	Y	Y

Ending Position

Y	Y	Y		X	X	X

Hoppy

Mathematics of the Game

As the number of markers in the game grows, the number of moves required to interchange their places grows as well. The challenge for students is twofold:

1. Exchange the markers in the minimum number of moves at each step.
2. Predict the number of moves needed as the number of markers gets larger.

In other words, students are looking for a pattern. They should make a t-table with the information they have found, then use this to predict the number of moves needed for the next step.
This strategy may also be called Guess and Check, or Predict and Verify.

The ultimate goal is to find the function relating the number of X's and Y's to the number of moves, and analyze it.

Hoppy

T-table of Results

Quadratic form

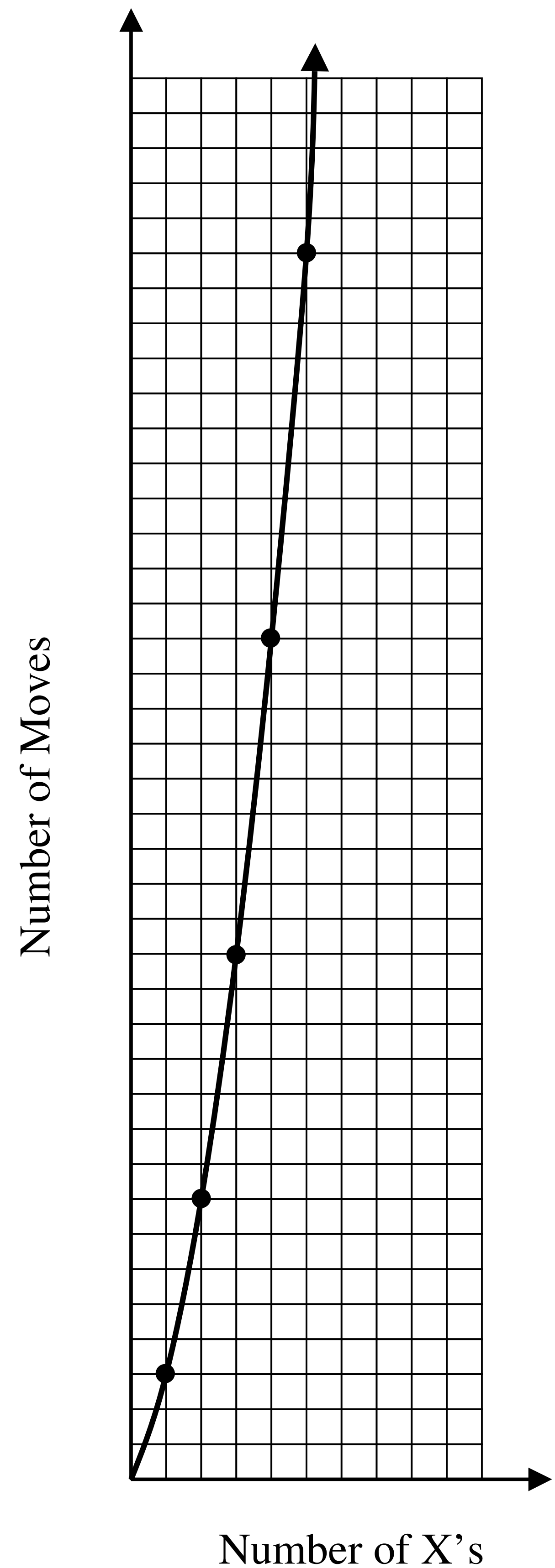
Number of X's and Y's	Number of moves needed
1	3
2	8
3	15
4	24
5	35
. . .	
n	$n(n + 2)$, or $n^2 + 2n$

Hoppy

Graph of Results

Parabolic shape

Number of X's and Y's	Number of moves needed
1	3
2	8
3	15
4	24
5	35
. . .	
n	n(n + 2), or $n^2 + 2n$



Hoppy 2

Extension – Unequal Markers

Put markers labeled *X* and *Y* as shown below. Your goal is to exchange their places with the fewest number of moves. Allowable moves:

- Sliding a marker onto an open space next to it.
- Jumping a marker over a single marker next to it, as long as you land on an empty space. You may not jump over more than one marker.

Starting Position

X	X		Y

Ending Position

Y		X	X

Hoppy

Standards Alignment – NCTM

All students should be able to:

- ♦ Build new mathematical knowledge through problem solving
- ♦ Solve problems that arise in mathematics and in other contexts
- ♦ Apply and adapt a variety of appropriate strategies to solve problems
- ♦ Monitor and reflect on the process of mathematical problem solving
- ♦ Make and investigate mathematical conjectures
- ♦ Understand patterns, relations, and functions
- ♦ Represent and analyze mathematical situations and structures using algebraic symbols
- ♦ Use mathematical models to represent and understand quantitative relationships
- ♦ Analyze and evaluate the mathematical thinking and strategies of others
- ♦ Use the language of mathematics to express mathematical ideas precisely

Hoppy

Standards Alignment – California Grade 7

Students shall be able to:

- ♦ Determine when and how to break a problem into simpler parts.
- ♦ Apply strategies and results from simpler problems to more complex problems.
- ♦ Make and test conjectures by using both inductive and deductive reasoning.
- ♦ Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.
- ♦ Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.
- ♦ Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language.
- ♦ Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.

Hoppy

Standards Alignment – California Algebra 1

16.0 Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.

17.0 Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.

18.0 Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.

24.0 Students use and know simple aspects of a logical argument:

24.1 Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.

Real World Number Stumpers

7 =	D in a W
26 =	L of the A
50 =	S in the U.S.
4 =	H on M R
3 =	W on a T
12 =	E in a D
18 =	H on a G C
13 =	S on the A F
52 =	C in a D
200=	D for P G in M
3 =	H in a B B
88 =	K on a P
3 =	BM (SHTR)
366 =	D in a LY
29 =	D in F in a LY

days in a week, letters of the alphabet, states in the United States, heads on Mount Rushmore, wheels on a tricycle, eggs in a dozen, holes on a golf course, stripes on the American flag, cards in a deck, dollars for passing Go in Monopoly, holes in a bowling ball, keys on a piano, blind mice (see how they run), days in a leap year, days in February in a leap year

Crossing the River

A group of 2 adults and 2 children want to cross a river. They have a small boat that can hold either:

one child or two children or one adult

The boat sinks with two adults, or one adult and one child. There must be at least one person in the boat when crossing the river (you can't pull the boat across the river with a rope).

1. How many trips will it take for all four people to cross the river? _____

2. If there are 3 adults and 2 children, how many trips will it take? ______

3. If there are 4 adults and 2 children, how many trips will it take? ______

4. Each time one adult joins the group, how many more trips will it take to cross the river? ______

5. Summarize your results in a t-table with a formula.

Number of Adults	Number of Trips
0	
1	
2	
3	
4	
5	
...	
x	

6. Explain how you determined your answer.

Crossing the River Manipulative Aid

Use these letter grids to assist students. Students may cut them out and move the letters back and forth as they arrive at a solution.

C	C	A	A	A	A	A	A
C	C	A	A	A	A	A	A
C	C	A	A	A	A	A	A
C	C	A	A	A	A	A	A
C	C	A	A	A	A	A	A
C	C	A	A	A	A	A	A
C	C	A	A	A	A	A	A
C	C	A	A	A	A	A	A

Crossing the River Manipulative Solution

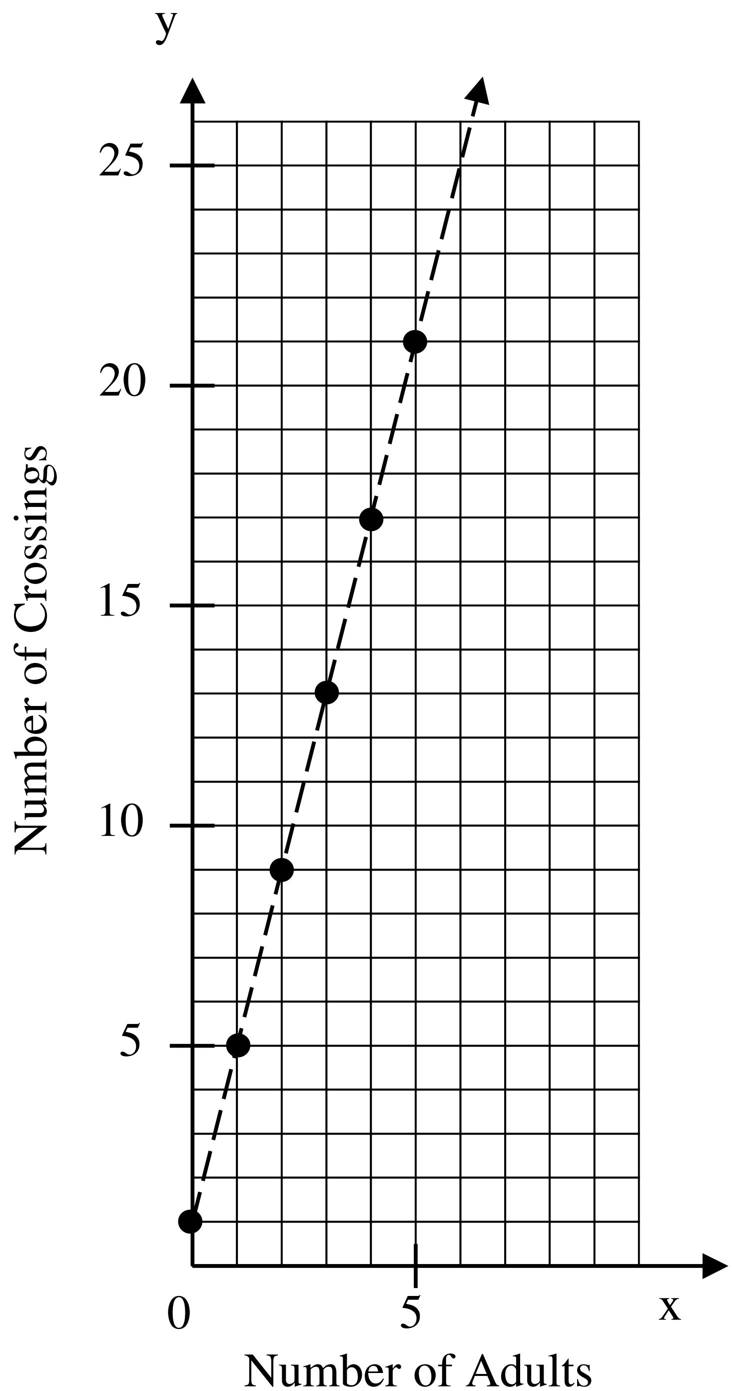
Result	Trip # and passengers	Result
A A C C		
A A	1 C C →	C C
A A C	2 C ←	C
A C	3 A →	A C
A C C	4 C ←	A
A	5 C C →	A C C
A C	6 C ←	A C
C	7 A →	A A C
C C	8 C ←	A A
	9 C C →	A A C C

Crossing the River
Graphical Solution

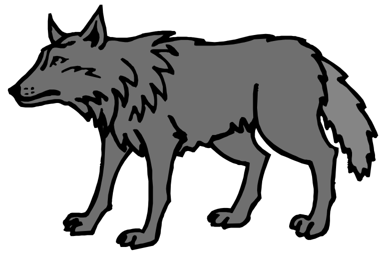
Two children can cross in one crossing. This is the y-intercept of the function.
Each adult added causes four more crossings.
This is the slope of the function.
Therefore,

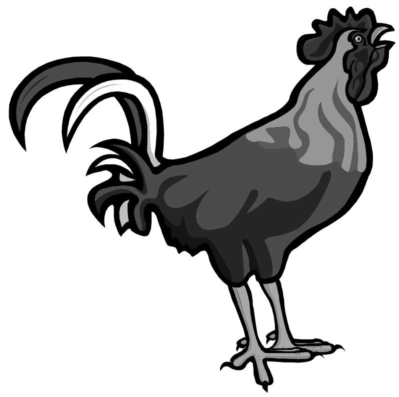
$$y = 4x + 1$$

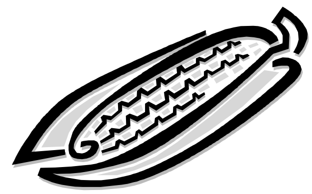
is the formula that describes the problem.



The Wolf, the Rooster, and the Corn





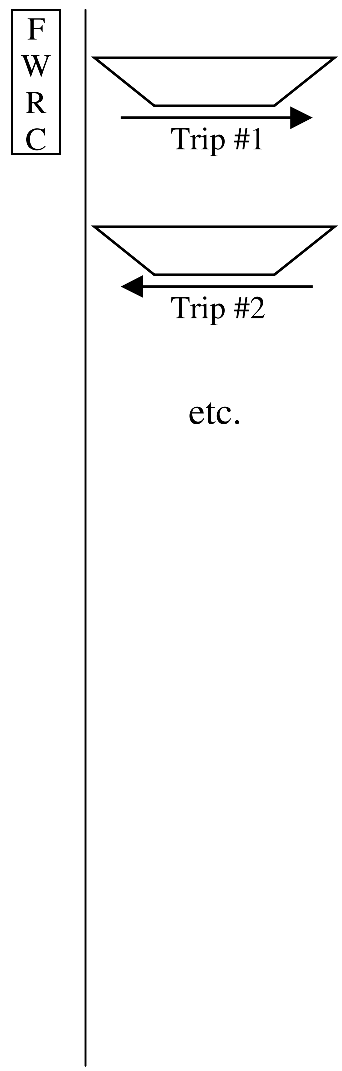


A farmer has a wolf, a rooster, and some corn, and he wants to cross a river with them. If he leaves the rooster alone with the wolf, the wolf will eat the rooster. If he leaves the corn alone with the rooster, the rooster will eat the corn. There is a small boat that can only carry the farmer and either the wolf, rooster, or corn. How does the farmer take everything across the river safely?

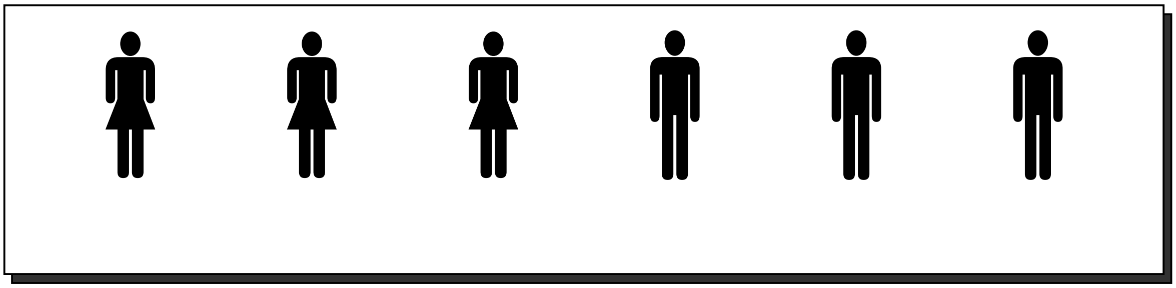
Help the farmer by giving him your plan to move everything to the other side of the river. Use the diagram at the right to sketch your solution for him. Show:

1. The occupants of the boat on each trip, and
2. Who/what is on each shore after each trip.

F = farmer
W = wolf
R = rooster
C = corn



The Three Wives and Their Jealous Husbands



Three wives and their husbands wish to cross a river. There is a small boat that can carry only two people at one time. Also, the husbands are very jealous and don't want their wives to be in the presence of another husband unless they are also present. How can all six of these folks cross the river safely?

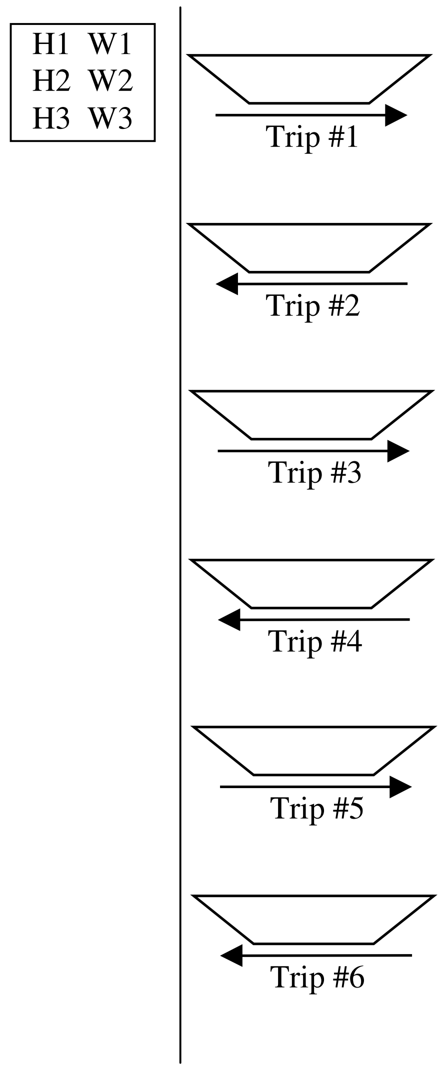
Help these couples by giving them your plan to move everyone to the other side of the river. Use the diagram at the right to sketch your solution. Show:

1. The occupants of the boat on each trip, and
2. Who is on each shore after each trip.

H1 = husband #1; W1 = wife #1

H2 = husband #2; W2 = wife #2

H3 = husband #3; W3 = wife #3



etc.

Sudoku: an Introduction

	1				
5	3		6		
1	2				
				1	5
		1		2	4
				3	

3			
4			1
1			4
			3

The rule:
Each row, column, and box must contain all the digits used.

		1	3
		2	
	2		
3	1		

				9				
		3	6		1	4		
1		8	4		2	5		9
2	1			4			7	5
6		5				8		2
7	8			5			1	3
5		2	3		7	1		8
		7	5		4	2		
				2				

ACTIVITY 12

Backwards Math

Materials:

- ☒ paper
- ☐ calculators
- ☐ transparency of activity master
- ☐ butcher paper

Overview:

This activity provides an excellent format for practicing operations. Students will not only work intently for long periods of time, they will want to learn even more advanced operations such as exponents, roots, and factorials. Classes and groups of students will be challenged to work together to explore and complete this task.

Vocabulary: operations

PROCEDURE

Skills:

- Performing operations
- Working with order of operations
- Problem solving

Good Tip!

If you teach more than one math class, they can compete against one another. Assign fours to one class, fives to another, sixes to the next and so on.

1. Tell the students that today's math will be backwards. You will give them all the answers. (This will usually make them pretty happy.) However, tell them that they must come up with the correct problem to the answer. In addition, the only numbers they can use are four 4's.

2. For example, they can get the answer one in these ways:

$^{4}/_{4} \times {}^{4}/_{4} = 1$ $\qquad$ $44 \div 44 = 1$

Challenge students to find other ways to make a problem equal to one.

3. Challenge students to find other ways to make a problem equal to one.

4. Next have them try to get the numbers one through ten. This will give you an opportunity to introduce the order of operations:

   a. Do work inside parentheses.
   b. Solve **exponents**.
   c. **Multiply** and **divide** from left to right in the problem.
   d. **Add** and **subtract** from left to right.

   For example, in this problem, solving correctly will result in an answer of 32:

   $$4^4/(4 + 4) = 256/(4 + 4) = 256/8 = 32$$

An acronym for the order of operations is P.E.M.D.A.S.: "Please Excuse My Dear Aunt Sally."

5. Have the students number a piece of paper one through one hundred (or whatever range you desire). Allow students to

make up problems and write them on the paper by the correct answers. Share these with the whole class to get them started. You may wish to have students work in teams of three or four.

6. For more advanced classes, you can introduce exponents, roots, factorials, and the greatest integer function. Some examples of these are given below.

Exponents	4^4 = 4 x 4 x 4 x 4= 256
Square roots	$\sqrt{4}$ = 2
Factorial	4! = 4 x 3 x 2 x 1 = 24
Greatest integer function	[4 x 4.44] = [17.76] = 17

Journal Prompts:

If you erased the four 4's in a problem and substituted four 5's, would any of them have the same answer? Why or why not?
Would four 9's be easier or more difficult to use? Why?
How many problems can you write that will have a solution of 1?

Homework:

You can ask students to finish the chart from one to one hundred. This will take more than one day. Even advanced students will have difficulty finding all 100 answers in a week. You might prefer to ask them to do ten problems per night.

Taking a Closer Look:

This assignment can be repeated using four fives, four sixes, and so on. Some numbers will provide greater difficulty.
For example, while 4^4 is usable in some problems, 8^8 rarely is since it is too large a number.
However $[8^{.8}]$ = [5.2780316] = 5.

Assessment:

Students can volunteer solutions to various answers. Write them on the board and have the class verify them. Those that work can stay on the list or chart. You may wish to include multiple solutions to some answers.

Good Tip!

For easy assessment, have students work in teams. One partner verifies the solution of another student. If they believe it's correct, they record it on the chart and they are given a point. If another team proves them wrong, that team gets two points and the incorrect solution is erased.

Backwards Math

Name ____________________________

Here are the answers to one hundred math problems. Use four 4's to create problems that will give these answers. Remember to use the correct order of operations to solve your problems: Parentheses, Exponents, Multiply or Divide, Add or Subtract.

1 = ______________________________
2 = ______________________________
3 = ______________________________
4 = ______________________________
5 = ______________________________
6 = ______________________________
7 = ______________________________
8 = ______________________________
9 = ______________________________
10 = ______________________________
11 = ______________________________
12 = ______________________________
13 = ______________________________
14 = ______________________________
15 = ______________________________
16 = ______________________________
17 = ______________________________
18 = ______________________________
19 = ______________________________
20 = ______________________________
21 = ______________________________
22 = ______________________________
23 = ______________________________
24 = ______________________________
25 = ______________________________
26 = ______________________________
27 = ______________________________
28 = ______________________________
29 = ______________________________
30 = ______________________________
31 = ______________________________
32 = ______________________________
33 = ______________________________
34 = ______________________________
35 = ______________________________
36 = ______________________________
37 = ______________________________
38 = ______________________________
39 = ______________________________
40 = ______________________________
41 = ______________________________
42 = ______________________________
43 = ______________________________
44 = ______________________________
45 = ______________________________
46 = ______________________________
47 = ______________________________
48 = ______________________________
49 = ______________________________
50 = ______________________________

from *Simply Great Math Activities: Number Sense*

Backwards Math

Name ____________________________

Here are the answers to one hundred math problems. Use four 4's to create problems that will give these answers. Remember to use the correct order of operations to solve your problems: Parentheses, Exponents, Multiply or Divide, Add or Subtract.

51 = ______________________________
52 = ______________________________
53 = ______________________________
54 = ______________________________
55 = ______________________________
56 = ______________________________
57 = ______________________________
58 = ______________________________
59 = ______________________________
60 = ______________________________
61 = ______________________________
62 = ______________________________
63 = ______________________________
64 = ______________________________
65 = ______________________________
66 = ______________________________
67 = ______________________________
68 = ______________________________
69 = ______________________________
70 = ______________________________
71 = ______________________________
72 = ______________________________
73 = ______________________________
74 = ______________________________
75 = ______________________________
76 = ______________________________
77 = ______________________________
78 = ______________________________
79 = ______________________________
80 = ______________________________
81 = ______________________________
82 = ______________________________
83 = ______________________________
84 = ______________________________
85 = ______________________________
86 = ______________________________
87 = ______________________________
88 = ______________________________
89 = ______________________________
90 = ______________________________
91 = ______________________________
92 = ______________________________
93 = ______________________________
94 = ______________________________
95 = ______________________________
96 = ______________________________
97 = ______________________________
98 = ______________________________
99 = ______________________________
100 = ______________________________

from *Simply Great Math Activities: Number Sense*

Backwards Math

Extensions and Ideas

Using four 4's, how many ways can you make an expression that equals one?

Then find as many ways as you can to make expressions equalling two, or three, etc.

Using the numbers 1, 2, 3, and 4, how many natural numbers from 1 to 100 can you make?

Then try this using 2, 3, 4, and 6, etc.

Using only <u>three</u> 4's, how many natural numbers in a row starting with one can you make?

Using the first four prime numbers: 2, 3, 5, 7, how many natural numbers from 1 to 100 can you make?

Using four 5's (or four 6's, etc.) how many natural numbers from 1 to 100 can you make?

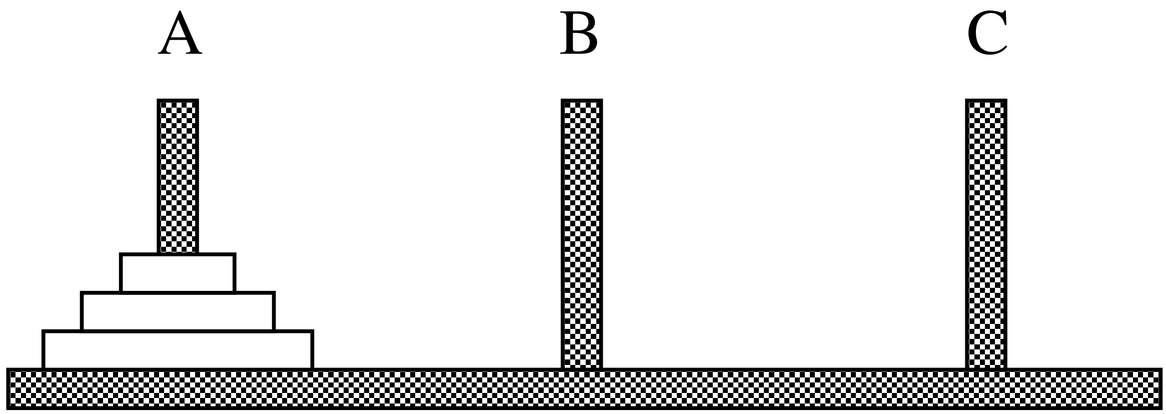
Common Themes – Algebra

What do these algebra expressions have in common?

1. $2(x - 4)$, $-3(y + 5)$, $-(6y + 2)$, $(w + 5)3$, . . .
2. 1^7, $5x/5x$, x^0,
3. $6/0$, $\sqrt{-16}$, 0^0, $\log(-3)$,
4. $|\ \ |$, $\sqrt{\ }$, $\leq$, π, θ, ∞
5. $x + 0 = 0$, $1 \cdot x = x$, $-1x = -x$, $x + (-x) = 0$, . .
6. $7x$ & $9x$, $-2y^2$ & $8y^2$, $4wz$ & $9wz$, . . .
7. bh, $a^2 + b^2 = c^2$, $2l + 2w$, πr^2, . . .
8. linear, quadratic, cubic, . . .
9. $(2, 3)$, $(-4, 6)$, $(5, 0)$, $(7, -3)$, . . .
10. $(\quad)$, $[\quad]$, $\{\quad\}$, $\overline{\qquad}$, . . .

distributive property, all equal one, undefined, math symbols, properties of real number system, like terms, formulas, types of equations, ordered pairs, symbols of inclusion

Towers of Hanoi



The tower of disks must be moved to a different peg and must end up in the same arrangement.

How many moves will it take?

There are only two rules:

- Disks must be moved one at a time
- A larger disk cannot be set on a smaller disk

Towers of Hanoi

Number of Disks	Number of Moves
1	
2	
3	
4	
5	
6	
. . .	
n	

Common Themes – Numbers

What do these numbers have in common?

1. 2, 56, 84, 12, 18, 2000 . . .
2. 7, 11, 19, 23, 2, 89 . . .
3. 1, 2, 3, 4, 6, 8, 12, 24
4. 13, 8, 34, 1, 3, 21, 55, 2 . . .
5. 0, 64, 16, 36, 49, 25, 9 . . .
6. 8, 1, 27, 125, 64, 216 . . .
7. 84, 35, 63, 14, 91, 21 . . .
8. 20, 220, 85, 90, 60, 45 . . .
9. 9, 22, 101, 49194, 101, 393 . . .
10. 24, 28, 84, 8, 100, 56 . . .

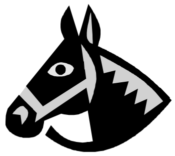
even numbers, primes, factors of 24, Fibonacci numbers, square numbers, cubic numbers, multiples of 7, multiples of 5, palindromes, multiples of 4

A Square for a Knight

Here is part of a magic square made by Leonhard Euler. Each row and column totals 260. Each half-row and half-column totals 260.

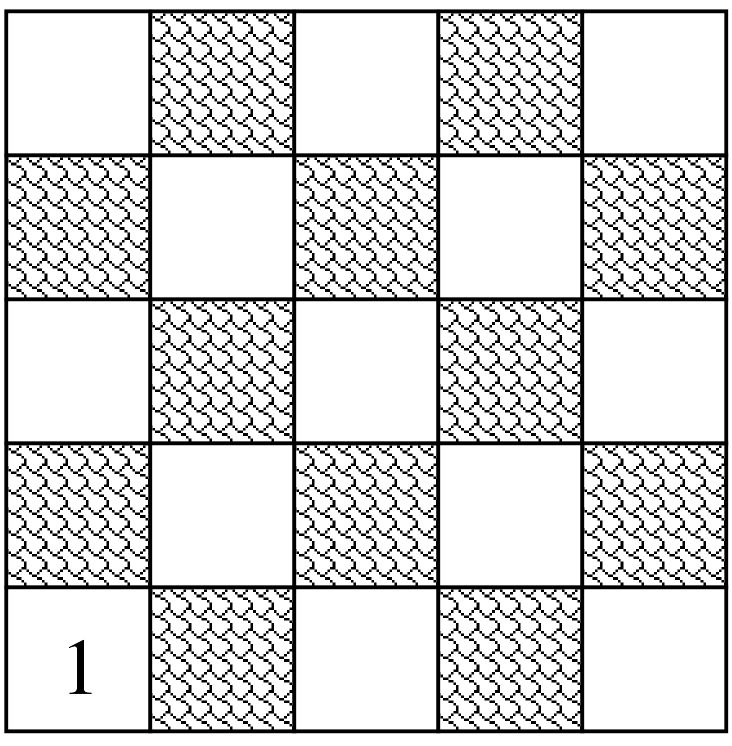
A chess knight, starting its L-shaped moves from box 1, can occupy all 64 squares in numerical order. This is called a Knight's Tour. Can you fill in the remaining squares?

1							
			3	62		14	
	2		32			17	
	29	4		20	61	36	
5			56				
		8	41		57	12	
					10		22



A Knight's Tour

It is possible to make a Knight's Tour of a 5x5 square. Starting in the square marked "1", proceed by making knight moves and number each visited square in order, until you reach the 25 th square.



Line by Line
A Mathematical Poem

1
1 1
2 1
1 2 1 1
1 1 1 2 2 1
3 1 2 2 1 1
1 3 1 1 2 2 2 1

Are you a poet and you don't know it?

Can you figure out the next line?

Visit the Teacher to Teacher Press website at

www.tttpress.com

On our website you will find:

- ♦ A complete catalog of our materials
- ♦ Free sample chapters from our books
- ♦ Downloadable handouts from workshops
- ♦ Quotes for motivating students
- ♦ Links to other valuable resource websites
- ♦ Order forms for our materials
- ♦ A bibliography of great math reading

Happy surfing!

Brad Fulton and Bill Lombard

Teacher to Teacher Press

"Building Mathematical Skill on a Foundation of Understanding"

Brad Fulton

- **Consultants**
- **Educators**
- **Authors**
- **Seminar leaders**
- **Teacher trainers**
- **Conference speakers**

Bill Lombard

PO Box 233, Millville, CA 96062
(530) 547-4687 brad@tttpress.com

5885 Avery Way, Redding, CA 96003
(530) 243-2064 bill@tttpress.com

Known throughout the country for motivating and engaging teachers and students, Brad and Bill have authored over ten books that provide easy-to-teach yet mathematically-rich activities for busy teachers. In addition, they have co-authored six teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

Seminar leaders and trainers of mathematics teachers

- California Math Council and NCTM presenters
- Lead trainers for summer teacher training institutes
- Trainers/consultants for district, county, regional, and national workshops

Authors and co-authors of mathematics curriculum

- *Simply Great Math Activities* series: five books covering all major strands
- *Math Discoveries* series: bringing math alive for students in middle schools
- Teacher training seminar materials handbooks for elementary, middle, and secondary school

Available for workshops, keynote addresses, and conference sessions

All workshops provide participants with complete and ready-to-use activities. These activities require minimal preparation, use materials commonly found in classrooms, and give clear and specific directions and format. Participants will also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

Brad and Bill's math activities are the best I've seen in 30 years of teaching!
Wayne Dequer, 7th grade math teacher

"The high-energy, easy-to-follow handouts were clear. The instructors were great!"
DeLinda Van Dyke, middle school teacher

References and sources for Pick's Theorem, Pentominoes, and Dot Paper

Solomon W. Golomb. Polyominoes – Puzzles, Patterns, Problems, and Packings. Princeton Science Library, 1994

~~Averbach, Bonnie, and Chein, Orin. *Problem Solving through Recreational Mathematics*. Dover Publications, 1980~~

~~Danesi, Marcel. *The Puzzle Instinct*. Indiana University Press, 2002~~

~~Gardner, Martin. *Mathematical Magic Show*. Alfred A. Knopf, 1977~~

~~Gardner, Martin. *My Best Mathematical and Logic Puzzles*. Dover Publications, 1994~~

~~Gardner, Martin. *Perplexing Puzzles and Tantalizing Teasers*. Dover Publications, 1988~~

~~Gardner, Martin. *Scientific American Book of Mathematical Puzzles & Diversions*. Simon and Schuster, 1961~~

~~Gardner, Martin. *Wheels, Life and other Mathematical Amusements*. W. H. Freeman and Company, 1999~~

~~Herr, Ted, and Johnson, Ken. *Problem Solving Strategies – Crossing the River with Dogs*. Key Curriculum Press, 1994~~

~~Holt, Michael. *Math Puzzles & Games*. Dorset Press, 1992~~

Lane County Mathematics Project. *Problem Solving in Mathematics* (several grade levels). Dale Seymour Publications, 1983

National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics.* NCTM, 2000

Polya, George. *How to Solve It*. Doubleday & Company, 1957

Polya, George. *Mathematical Discovery, volumes 1 and 2*. John Wiley & Sons, 1962